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Amount of Correlation Function-based Diversity Motivation Design in MIMO Systems

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(Abstract) Improve the reliability in terms of diversity in wireless communications using MIMO systems is crucial as MIMO channels is spatially and temporally correlated. The impact of correlations on space-time codes results in the decrease of their performance in terms of bit-error rate (BER). Hence, it is reasonable to seek for metrics which provide the measure of fading correlations. In This paper we investigate the measure of correlation in spatial domain by using the amount of correlation as a metric. High (resp. low) amount of correlation value means low (resp. high) diversity and therefore the knowledge of the variation of this metric lets to foresee the performance analysis of MIMO channels. By considering the one-ring scattering model as the wireless communication Channel, we first compute the amount of correlation function and then show that a wise tradeoff between parameters can be envisaged in order to minimize MIMO channel correlation. AS application, MIMO system with two transmit antenna and two receive antenna (2x2 MIMO) with orthogonal space-time block (OSTBC) are implemented with high reliability.

Keywords: MIMO; OSTBC; One-ring scattering; Diversity; Amount of Correlation.

1. INTRODUCTION

MIMO systems with space-time coding in comparison with Single-input single-output (SISO) systems are known to be increasing the capacity (Shannon capacity) and the performance (bit-error rate) of the link of communication .

However, one of the major drawbacks of the MIMO system is that its channel is often correlated. Correlations are often generated from antenna spacing and scattering object surrounding the base station (bs) or the mobile station (ms)[1].As its name suggests ,correlation means relation with, and thus, fading correlation between two arbitrary MIMO sub channels lead to the decreasing of the degree of freedom or the diversity .This contributes to impact on the space-time codes by decreasing their performance in terms of bit-error rate [2].

Another significant drawback of a MIMO system can be the estimation of its high number of parameters required in the full correlation matrix of the channel [3].Therefore, it is reasonable to seek for correlation metrics which not only may allow an interpretation of each individual correlation parameter due to the fact that a such alternative becomes difficult, but also to set up metrics which can take into account all of the parameters.

One method to measure the correlations in MIMO channel is to quantify the amount of correlation defined in [4]. Fading correlations are directly related to the diversity gain as a high (resp. low) amount of correlation value induces low (resp. high) diversity.

Assuming the one-ring scattering MIMO channel system , we adopt the above-mentioned method and we show that a wise tradeoff between parameters can be done in order to minimize significantly the amount of correlation .As application, a MIMO system with two transmit antenna and two receive antenna with OSTBC signaling is implemented with high reliability

The remainder of the paper is organized as follows: the section 2 defines the system model; section 3 provides a general theoretical analysis of the amount of correlation of MIMO systems. In Section 4, we provide the simulation .In section 5, we summarize the study.

2. SYSTEM MODEL

Consider a MIMO channel H with n_t transmit and n_r received antennas as illustrated in figure 1 The correlation $r_{kl,qs}(\Delta\tau)$ between two arbitrary MIMO sub channels kl and qs with a delay difference $\Delta\tau$, is confined in a $p\times p$ correlation matrix R defined :

$$\mathrm{R} = \varepsilon\left\{\mathrm{vec(H)}\,\mathrm{vec(H)}^{H}\right\} \qquad (1)$$

ε denotes the expectation and $(.)^{H}$ the conjugate transpose and $P{=}n_{t\times}n_{r}$.

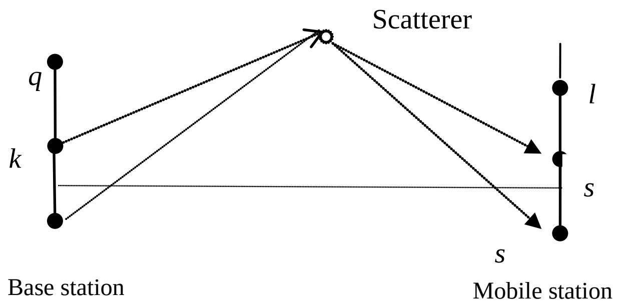

Figure .1 MIMO Scattering channel

.For the reader understanding, it should be noted that waves emitted from the transceiver undergo scattering environments. Notice also a MIMO sub channel can be the link joining a k^{th} antenna at the base station and a l^{th} antenna at the mobile station.

q, k and *s, l* are integer numbers and can take values respectively from $[1, n_t]$ and $[1, n_r]$.All combinations are feasible and $r_{kl,qs}(\Delta\tau)$ can denote a transmit correlation , a receive correlation or a cross correlation, by setting up respectively $l \neq s$,$k=q$; $l=s$, $k\neq q$, and $l \neq s$,$k \neq q$.

It can be seen that the major drawback of the full correlation matrix R is its huge size. Its requires a number of $p2$ elements $r_{kl,qs}(\Delta\tau)$ to be fully estimated with respect to the propagation environments (scattering environments) of electromagnetic waves.

The amount of correlation of a MIMO channel lightly modified in [4] is defined as:

$$\psi_p^{\Delta\tau} = \sqrt{\frac{1}{p(p-1)}\sum_{i=1}^{p}\sum_{\substack{j=1\\ j\neq 1}}^{p}\left|r_{kl,qs}(\Delta\tau)\right|^2} \qquad (2)$$

$$P = n_{t\times} n_r$$

$$q = [1, n_t];\ q = [1, n_t];$$
$$l = [1, n_r];\ s = [1, n_r];$$

In **Eq.2**, $\psi_p^{\Delta\tau}$ can simply be interpreted as an average on all the parameters of the correlation matrix R. It also defines the performance in terms of correlations in the channel.

For instance, $\psi_p^{\Delta\tau} = 1$ indicates high correlation or *degenerate channel*[5] while $\psi_p^{\Delta\tau} = 0$ defines an uncorrelated channel .Therefore a wise tradeoff between factors can be performed in order to minimize the amount of correlation. However, it will return to analyze factors that contribute to high correlation or act to minimize correlation. This will return also to estimate $r_{kl,qs}(\Delta\tau)$ which depend rather on MIMO scattering channel modeling.

The next discussion will focus on the evaluation of $\psi_p^{\Delta\tau}$ assuming the One Ring Scattering MIMO Channel

3. ONE RING SCATTERING MIMO CHANNEL

In this section, we will perform the computation of the amount of correlation of MIMO channels according to **Eq**.2. Without losing any generality a 2x2 MIMO configuration will be used as template to illustrate some proprieties of the elements of the matrix R. For instance, for 2x2 MIMO system, the 4x4 correlation matrix R can be simply written in terms of transmit ,receive and cross correlations as follows:

$$R = \begin{pmatrix} 1 & r_{11,12}(\Delta\tau) & r_{11,21}(\Delta\tau)^* & r_{11,22}(\Delta\tau)^* \\ r_{11,12}(\Delta\tau)^* & 1 & r_{12,21}(\Delta\tau)^* & r_{12,22}(\Delta\tau)^* \\ r_{11,21}(\Delta\tau) & r_{12,21}(\Delta\tau) & 1 & r_{21,22}(\Delta\tau)^* \\ r_{11,22}(\Delta\tau) & r_{12,22}(\Delta\tau) & r_{21,22}(\Delta\tau) & 1 \end{pmatrix}.$$

where :

- $r_{11,12}(\Delta\tau)$, $r_{21,22}(\Delta\tau)$ are respectively transmit correlations seen from antenna 1and antenna 2 at the receive array
- $r_{11,21}(\Delta\tau)$, $r_{12,22}(\Delta\tau)$ are respectively receive correlations seen at antenna 1 and antenna2 at the transmit array.
- $r_{11,22}(\Delta\tau)$, $r_{12,21}(\Delta\tau)$ are cross-correlations

As it has already mentioned, the strict analysis of the matrix R will turn to a hard study of P^2(p=4 for 2x2 MIMO systems) parameters with respect of physical propagation of the radio channel. However, we will consider that both transmit antennas and receive antennas have the same radiation pattern and the same orientation; then we can write as in [6], correlation parameters like:

$$\begin{aligned} r_{11,12}(\Delta\tau) &= r_{21,22}(\Delta\tau) \\ r_{11,21}(\Delta\tau) &= r_{12,22}(\Delta\tau) \\ r_{11,22}(\Delta\tau) &= r_{12,21}(\Delta\tau) \end{aligned} \qquad (3)$$

Therefore, the relationship in **Eq.3** reduceed by $1/p$ the number of parameters to be fully specified in the matrix R. Similar to **Eq. 3** and by extension, the same analysis can be done for any MIMO system with n_t transmit and n_r receive antenna .

3.1. One-ring Scattering Model

The One-ring scattering environment model is a geometry-based model. It is often assumed that the base station is elevated and not surrounded by local scattering while the mobile station (MS) is obstructed by a ring of scatterers.
Assuming a vertical array of antenna at both receiver and transmitter (fig1) and $\Delta\tau = 0$, a arbitrary correlation parameters $r_{rk,sq}(0)$ can be formulated from its generic form in[7] :

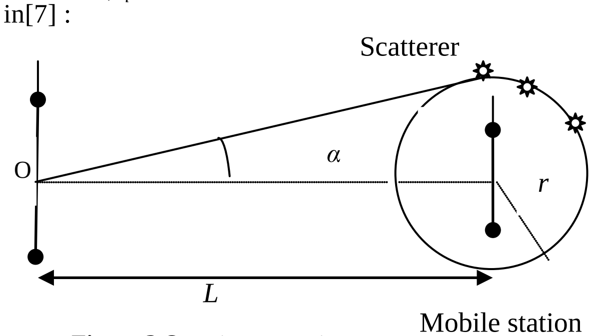

Figure.2 One-ring scattering

$$r_{lk,sq}(0)=\int_{-\pi}^{\pi} A_{lk}A^*_{sq}\,p(\theta)d\theta \qquad (4)$$

Where $p(\theta)$ denotes the probability distribution function (PDF)in the angular domain of scatterers surrounding the mobile.

$$A_{lk}=e^{j\pi(3-2l)\frac{d}{\lambda}\left[\alpha\sin(\theta)\right]}\times e^{j\pi(3-2k)\frac{D}{\lambda}\left[\cos\left(\theta-\frac{\pi}{2}\right)\right]}$$

A_{sq} is obtained by replacing k by q and l by s.
Assuming a uniform PDF($p(\theta)$ =1/π), from **Eq**.4 we derive:

$$r_{lk,sq}(0)=J_0\left(\frac{2\Pi}{\lambda}d(s-l)+\frac{2\Pi}{\lambda}\alpha D(q-k)\right) \quad (5)$$

under conditions the combination of s,l and q,k should be done such that $s\geq l$,$q\geq k$.
d, D, are respectively antenna space between two consecutive antennas in the receive array and in the transmit array. α depends on the environment of wave propagation i.e. the scattering ring size (the radius r of the ring) and the distance L between the base and the mobile; α =tang(r /L) . λ is the wavelength and $J_0(.)$ the zero order Bessel function of the first kind.
Setting $n_r=2$, $n_t=2$, we establish:

$$r_{11,12}(0)=J_0\left(\frac{2\Pi}{\lambda}d\right), r_{11,21}(0)=J_0\left(\frac{2\Pi}{\lambda}\alpha D\right)$$

$$r_{11,22}(0)=J_0\left(\frac{2\Pi}{\lambda}d+\frac{2\Pi}{\lambda}\alpha D\right) \qquad (6)$$

similar to [8]and $r_{11,12}(0)=r_{21,22}(0), r_{12,22}(0)=r_{11,21}(0)$ $r_{11,22}(0)=r_{12,21}(0)$ as predicted in **Eq.3**

3.2. Amount of Correlation

Let now investigate the generalization of **Eq.6** in to an arbitrary MIMO system.
Keeping in mind that s, l are integers and define positions or range occupied by any pair of antennas at the receive array, we can write the relation:

$s-l=i$ with i=0,1,2,….. n_r-1 (7)

like **Eq.7** we can establish the same relation for the transmit array:

$q-k=j$ with j=0,1,2,…… n_t-1 (8)

by replacing $s-l$ by i and q-k by j **,Eq.5** can be rewritten as:

$$r_{lk,sq}(0)=J_0\left(\frac{2\Pi}{\lambda}d.i+\frac{2\Pi}{\lambda}\alpha D.j\right) \qquad (9)$$

from where we can easily drawn the general form of the transmit, receive and cross correlation as:

$$r_{lk,sk}(0)=J_0\left(\frac{2\Pi}{\lambda}d.i\right), r_{lk,lq}(0)=J_0\left(\frac{2\Pi}{\lambda}\alpha D.j\right)$$

$$r_{lk,sq}(0)=J_0\left(\frac{2\Pi}{\lambda}d.i+\frac{2\Pi}{\lambda}\alpha D.j\right)$$

Keeping in mind that the number of parameters of the matrix R is reduced by 1/p, the amount of correlation from the combination of **Eq. 2**, and **Eq. 9** can be computed:

$$\psi_p^0=\frac{1}{\sqrt{p-1}}\begin{pmatrix}\sum_{i=1}^{n_t-1}J_0{}^2\left(\frac{2\pi}{\lambda}i.d\right)+\sum_{j=1}^{n_r-1}J_0{}^2\left(\frac{2\pi}{\lambda}jD\alpha\right)+\\ \sum_{i=1}^{n_t-1}\sum_{j=1}^{n_r-1}J_0{}^2\left(\frac{2\pi}{\lambda}i.d+\frac{2\pi}{\lambda}j.D\alpha\right)\end{pmatrix}^{1/2}$$

(10)

4.SIMULATION RESULT

In this section, we foresee the performance of the 2x2 MIMO one-ring scattering channel through its amount of correlation ψ_{2x2}^0 . .We will consider α varying because it depends on the scattering ring r and the distance L which vary in practical circumstances.

ψ_{2x2}^0 from **Eq.10** is computed:

$$\psi_{2x2}^0=\frac{1}{\sqrt{3}}\begin{pmatrix}J_0{}^2\left(\frac{2\pi}{\lambda}d\right)+J_0{}^2\left(\frac{2\pi}{\lambda}Dd\right)+\\ J_0{}^2\left(\frac{2\pi}{\lambda}d+\frac{2\pi}{\lambda}D\alpha\right)\end{pmatrix}^{1/2} \qquad (11)$$

Our aim is also to have the amount of ψ_{2x2}^0 correlation lowered as much as possible in order to be closed to that of uncorrelated channel. We will assume the frequency band of 5 GHz. The performance of the channel (**Figure**3) is simulated for different antennas spacing $d=k\lambda$ and $D=q\lambda$ and following beamwidth α . The amplitude for different cases decreases following the increasing of the beamwidth .Two types of channels(channel 1 and channel 2) derived from this result and characterized by their amount of correlation (see **Table 1**) will be used to simulate the performance of BER of OSTBC (Alamouti code) .
Assuming ML decoding and Quadrature Phase–Shift Keying (QPSK) as modulation scheme, the performances of the Alamouti code over channel1 (**Figure.4**) and channel2 (**Figure.5**) are compared with that of the independent and identical distributed channel(i.i.d) .

The precoding matrix in [9], P=*diag* [1, exp (j.0.93)] where *diag* means ‘diagonal’, has been applied in addition to our design. It can be seen that it does not almost change the performance of our design. This is due to the fact the two MIMO channels approach the uncorrelated channel. Mainly the performance of OSTBC over channel 2(Fig 3) reaches

roughly that over uncorrelated channel. For instance, at signal-to noise ratio (Eb/No) of 11db, the BER is10^{-3} in uncorrelated channel, and 1.3$10^{-3}$ in channel 2.

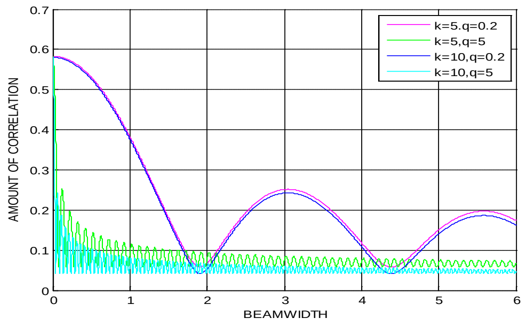

Figure.3 Amount of correlation for d=k λ and D=q λ

TABLE I. TYPES OF CHANNEL OF THE ONE-RING MODEL

	α	d	D	ψ^{0}_{2x2}
Channel	0.3542	10λ	5 λ	0.1
Channel	0.1763	10λ	5 λ	0.0412

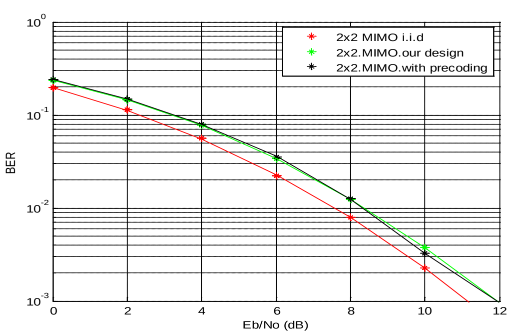

Figure. 4 Performance of OSTBC over channel 1

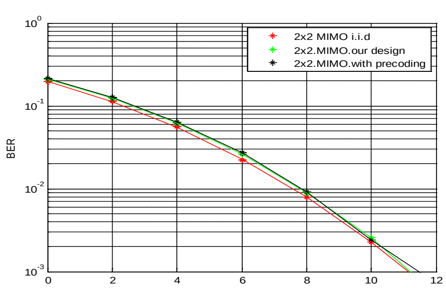

Figure .5 Performance of OSTBC over channel 2

5. CONCLUSION

We have investigated the correlation proprieties of wireless communications using MIMO systems through their amount of correlation. The one ring scattering channel model has been considered as the environment of wave propagation. For this model, our study reflected by the simulation result of the amount of correlation reveals the possibility to reach roughly the proprieties of i.i.d channel. The performance in terms of the bit error-rate of the orthogonal space time block code (Alamouti code) simulated over two types of 2x2 MIMO channel characterized by their amount of correlation and is shown with high reliability.

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